

Gaming with system to game the system!

Jake Quinn

Spring 2024

Introduction

As a kid, I often played games like Chopsticks or Tic-Tac-Toe. As I played them more, I started to discover ways to force draws. At the time, it was exciting to gradually come up with solutions to the games. In this seminar, we will try to reveal in ways to unwind more complicated games. In these logic-based, two-player games, a famous game theoretic result (Zermelo's Theorem) tells us that if the game cannot end in a draw, a player has a winning strategy. We will explore the consequences of this theorem (when it applies) by playing and strategizing in several two-player games over the quarter. The general format of the class will be in two week blocks:

Week I: We introduce and play the game (or two) of interest. In this week, you will have the opportunity to play the game against fellow classmates. As optional homework, you can think about winning strategies for the game (if a winning strategy exists) or play the game more with friends.

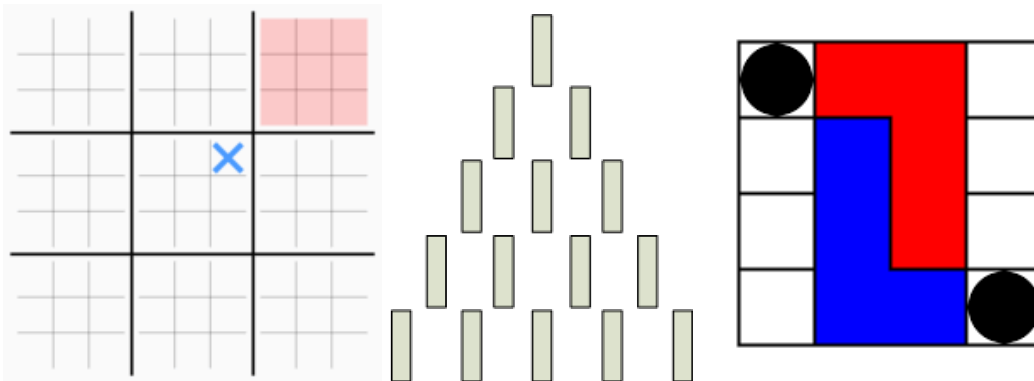
Week II: Students can share their findings about the game. We will start by forming small groups in which we can share our preliminary ideas. Then, we can come back together for a first brainstorm round. If productive, we can continue a class discussion for the rest of the session. If we need more time to think, we can return to brainstorming groups. The content goal will be to discuss whether or not there is a winning strategy. If there is, we will try to formulate winning strategies. If we can force a draw, we will try to formulate how.

Prerequisites

There are no prerequisites for this course!

Learning Objectives

1. Learn and play with strategies in two-player games (e.g. how binary numbers can help model the game).
2. Explore how one can force a particular outcome in a game (e.g. by exhausting a decision tree).
3. (If time allows) Think about ways we can improve a game if we find a solution to it to make the game more interesting (e.g. prohibiting placements in dead squares in Ultimate Tic-Tac-Toe).





Course Outline

- **Week 1:** Playing the game of Nim
- **Week 2:** Presenting findings on the game of Nim
- **Week 3:** Playing the game Ultimate Tic-Tac-Toe and the n pegs and m holes game
- **Week 4:** Presenting findings on the game Ultimate Tic-Tac-Toe and the n pegs and m holes game
- **Week 5:** Playing the L game and the Dots and Boxes game
- **Week 6:** Presenting findings on the L game and the Dots and Boxes game
- **Week 7:** Playing the game of Order and Chaos
- **Week 8:** Presenting findings on the game of Order and Chaos
- **Week 9:** Playing the game Connect Four
- **Week 10:** Presenting findings on the game Connect Four

Optional Project

Pick any finite two-person game of perfect information in which players alternate moves and chance does not affect the decision-making process (it could be one of the games played in class or a different one). Make a formal write-up about whether there is a winning strategy for the first or second player or whether it ends in a draw. Notice that some games cannot end in a draw (e.g. game of Order and Chaos, n pegs and m holes game). Hence, for these games Zermelo's theorem says there *must* be a winning strategy for one of the players. Other games can end in draws and yet still have winning strategies (e.g. Connect Four or Ultimate Tic-Tac-Toe). Still other games must end in draws if players play optimally (e.g. regular Tic-Tac-Toe).

I will give feedback on these projects if submitted.

Logistical Disclaimers

- I do *not* have all these games figured out. As such, we should not look to me as any sort of authority when it comes to the discussions. I will be there to moderate and provide suggestions, but I will probably hold back from sharing too much myself (*especially if I do know a solution*).
- If we do decide we wish to focus on a subset of the games (perhaps because we were having an interesting discussion on a particular week and wanted to continue), I am flexible and willing to alter the course outline.
- If there is a game you do not see here which you think is particularly relevant to the seminar's theme, you can discuss it with me independent of the seminar or we can propose the game to the class. If there is class agreement, we can edit the games listed in the course outline.